

# Estimating the True-Cost Index of the Piglog Model

The true-cost index for any household, within the context of the piglog model, may be written as:

$$\ln P(p_1, p_0; u)_{hR} = [a(p_1) - a(p_0)] + [b(p_1) - b(p_0)] u_{hR}, \quad (30)$$

for price vectors  $p_1$  and  $p_0$  and for reference utility  $u_{hR}$ . Equation 30 can be interpreted as the cost of living at a minimum level of consumer expenditure, say  $\ln S_t = a(p_1) - a(p_0)$ , and a marginal expenditure index,  $\ln M_t = [b(p_1) - b(p_0)] u_{hR}$ . Fry and Pashardes (1989) note that this interpretation is useful because changes in  $\ln S_t$  over time should incorporate the effects of substitution among goods, while differences in  $\ln M_t$  across households should reflect the distributional effects of inflation.

If one uses the associated indirect utility function  $u_h = [\ln x_h - a(p)]/b(p)$ , where  $x_h$  is the expenditure of the  $h^{th}$  household, the Marshallian budget shares of the piglog model can be derived as:

$$w_{iht} = a_i(p_t) + [b_i(p_t)/b(p_t)][\ln x_{ht} - a(p_t)]. \quad (31)$$

This complete demand system could be estimated, but one is generally limited in the number of commodities or groups that can be considered because of the effects of multicollinearity. Demand systems are usually limited to 8 to 12 different categories of goods. A high degree of aggregation generally results in little substitution between the groups. Rather, most of the substitution occurs within the separate groupings, and these substitution effects are lost in the estimation process. However, Fry and Pashardes (1989) propose a different strategy for dealing with a larger number of commodities. They propose modeling the substitution effects as shifts in the  $a(p)$  part of the piglog cost function over time.

Specifically, when the piglog cost function takes the Almost Ideal Demand System form, we can write the Engel curve as:

$$w_{iht} = A_{it} + B_i [\ln x_{ht} - a(p_t)], \quad (32)$$

where

$$A_{it} = A_{i0} + \sum_j \lambda_{ij} \ln(p_{jt}/p_{j0}).$$

The  $A_{it}$  terms reflect the substitution effects imbedded in the time-varying intercept as prices change from  $p_{i0}$ , and where  $a(p_t)$  is equal to the household with the minimum expenditure level.

The results of the estimation of the above Engel curves can be used to construct a base-period referenced true-cost index series for any given household  $h$  (Fry and Pashardes, 1989) as shown in the following equation:

$$\ln I_{ht} = \sum_r A_{r0} \ln(p_{it}/p_{i0}) \sum_r A_{rt} \ln(p_{it}/p_{i0}) + [\Pi_i(p_{it})^{\beta_i} - 1]. \quad (33)$$

We see that three indexes can be derived from estimation of the Engel curves. The first is a fixed-weight price index. The second is a price index that shows the effects of substitution. The third is a marginal index that shows the effect on the index of different expenditure levels. The average of the first two indexes is the reference household's true-cost index. It corresponds to the original Tornqvist index except that the  $A_{it}$  terms from the estimated Engel curves are used in place of the budget shares. These intercept terms reflect the substitution effects that occur over time as relative prices change. All other indexes are relative to the reference household's index and differ by the effect of their expenditure level.