The Piglog Model

The cost function underlying both the Engel curves and the Tornqvist type of indexes we constructed are based on the piglog functional form (Deaton and Muellbauer, 1980). The piglog model was developed to treat aggregate consumer behavior as if it were the outcome of a single maximizing consumer. This problem of how to treat aggregate consumer behavior as if it were the outcome of a single maximizing consumer exists because it is neither necessary nor desirable for a macroeconomic relationship to perfectly mimic its microeconomic foundation. Hence, in demand analysis, theory deals with behavior at the individual level and the laws of demand apply to individuals. At the micro level, conditions like symmetry and separability may hold, while at the macro level they may not. Therefore, the market demand functions that we estimate may not have the desirable properties of micro demand functions. This problem is known in economics as the aggregation problem.

We assume that each household's expenditure (x_h) is exogenous and that it varies from household to household. The prices of different goods are assumed to be the same for all consumers, and this assumption is crucial to the analysis. Its effect is to ensure that all consumers face the same prices so that only differences in expenditure levels need to be considered. Consequently, the conditions that must exist for aggregation can be determined by establishing what restrictions have to be placed on the Engel curve.

First, consider the individual demand for the i^{th} good in the h^{th} household, and write the demand function as:

$$q_{ih} = g_{ih} (x_h, p). \qquad (3)$$

If there are H different households, then average demand would imply that:

$$\bar{q} = f_i (x_1, x_2, ...x_h, p) = (\Sigma g_{ih} (x_h, p))/H, \qquad (4)$$

for some function f_i. Exact linear aggregation is possible if we can write the above equation as:

$$\bar{q} = g(\bar{x}, p). \qquad (5)$$

This last term implies that a reallocation of a single unit of currency from any one household to another must leave market demands unchanged. In other words, equation 5 implies that the marginal propensity to spend must be identical for all households. This, in turn, means that the above function must be linear in expenditure (x_h) for some functions α_{ih} and β_i of p alone, such that:

$$q_{ih} = \alpha_{ih} (p) + \beta_i (p)\, x_h, \qquad (6)$$

where α is indexed on h but β is not. Note that if either α or β were negative, then expenditures would have to be restricted within some range to keep quantities from also being negative. For the aggregate function, we would then have:

$$\bar{q} = \alpha_i (p) = \beta_i (p)\, \bar{x}, \qquad (7)$$

where it is assumed that none of the individual x_h's is such to make quantities negative. If, however, we do not want to place any restrictions on the range of expenditures, we must then delete the intercepts from the two preceding equations. This then implies that quantities demanded will be proportional to expenditures, which is a severe restriction.

If we go one step further and assume that our representative consumer maximizes utility (denoted u_h), then each household would have the cost function:

$$c_h(u_h,p) = a_h(p) + u_h(b(p)), \qquad (8)$$

where the corresponding average function would be:

$$\bar{x} = c(u,p) = a(\bar{p}) + ub(\bar{p}). \qquad (9)$$

Thus, if we assume that individuals maximize utility and preferences so as to satisfy the aggregation condition, the average demands above will also be consistent with utility maximization. This is desirable because Engel curves can be derived from the above cost function, equation 8.

Note that the cost function in equation 8 implies quasi-homothetic preferences, or linear Engel curves. This is a very strong restriction, as we showed earlier. For broad aggregate data, it is possible that all consumers will have some positive purchases. For disagregated data, however, there very likely would be some zero purchases, which would then require Engel curves without intercepts. As noted above, this then implies that quantities are proportional to expenditures.

Another approach to the problem that leads directly to a piglog model is to require exact nonlinear aggregation. One difference with this approach is that average budget shares are used as the dependent variable. Hence, we define the average budget share for the i^{th} good as:

$$\bar{w}_i = p_i \Sigma_h q_{ih} / \Sigma_h x_h = \Sigma_h (x_h / \Sigma x_h)\, w_{ih}, \qquad (10)$$

so that the market demand is a weighted average of individual household demands, the weights being proportional to the expenditure of each household.

If we restrict the average budget share to be a function of prices and average expenditure, we arrive at the same results as before: linear Engel curves. However, nonlinear aggregation requires that average budget shares depend on prices and a representative level of expenditure. Hence, the market demand can be thought of as deriving from the behavior of a single representative consumer faced with prices p and expenditure x_0.

A representative consumer exists if some indirect utility function $\varphi(x,\mathrm{p})$, with corresponding cost function $c(u,p)$ exists, so that for some level of utility $u_0 = \varphi(x_0,p)$, we have:

$$\begin{aligned} \bar{w}_i &= w_i(u_0,p) = \delta \ln c(u_0,p)/\delta \ln p_i \qquad (11) \\ &= \Sigma\, (x_h/\Sigma x_h)\ \delta \ln c_h(u_h,p)/\delta \ln p, \end{aligned}$$

where $c_h(u_0,p)$ is the cost function of household h, with $u_h = \varphi(x_h,p)$. The cost function from which the above average budget share equation can be derived must take the form:

$$c_h(u_0,p) = \theta[u_0,\, a(p),\, b(p)] + \phi_h(p), \qquad (12)$$

where $a(p)$, $b(p)$, and $\varphi_h(p)$ are linearly homogeneous functions of prices and θ is linearly homogeneous in a and b. Over all consumers, the $\theta_h(p)$ functions must sum to zero, so that the representative cost function takes the form:

$$c(u_0,p) = \theta[u_0,\, a(p),\, b(p)], \qquad (13)$$

for the same functions $a(p)$ and $b(p)$. These two functions can be thought of as the prices of two intermediate goods that, together with utility, define the macro cost function. From this cost function, the representative average budget can be derived as:

$$\begin{aligned} \bar{w}_i &= (\delta 1\mathrm{n}\, \theta/\delta 1\mathrm{n}\, a)\, (\delta 1\mathrm{n}\, a/\delta 1\mathrm{n}\, p_i) \qquad (14) \\ &\quad + (\delta 1\mathrm{n}\, \theta/\delta 1\mathrm{n}\, b)\, (\delta 1\mathrm{n}\, b/\delta 1\mathrm{n}\, p_i). \end{aligned}$$

Since θ is homogeneous of degree one, the above equation can be written as:

$$\bar{w}_i = (1 - \lambda)\ \delta \ln a/\delta \ln p_i + \lambda\ \delta \ln b/\delta \ln p_i, \qquad (15)$$

where

$$\lambda = \delta \ln \theta / \delta \ln b = \lambda(x_0, p).$$

Thus, each budget share is a weighted sum of the value shares associated with the two functions $a(p)$ and $b(p)$, with the weights depending on representative utility, u_0, or total expenditure. In addition, prices are the same for all consumers. Consequently, at constant prices, each budget share is a linear function of all other budget shares. This cost function still places strong restrictions on the Engel curves. For instance, the slopes of the Engel curves representing different households vary linearly with one another as total expenditures change at constant prices. This does not mean that the Engel curves are linear themselves.

The cost function in equation 13 allows for consistent nonlinear aggregation. By definition, representative expenditure x_0 will be some point in the expenditure distribution, the position of which is determined by the degree of nonlinearity of the Engel curves and by the price vector p. When the representative expenditure level is independent of prices and depends only on the distribution of expenditures, we have what is known as price-independent generalized linearity (pigl). Its general cost function is given by:

$$c_h(u_h, p) = k_h[a(p)^{\alpha}(1 - u_h) + b(p)^{\alpha} u_h]^{1/\alpha}, \tag{16}$$

with a representative cost function of:

$$c(u_0, p) = [a(p)^{\alpha}(1 - u_0) + b(p)^{\alpha} u_0]^{1/\alpha}. \tag{17}$$

When α tends to zero, we have the piglog model where:

$$\ln c(u_0, p) = (1 - u_0) \ln a(p) + u_0 \ln (p). \tag{18}$$

The nonlinear Engel curve associated with this cost function is:

$$w_i = \gamma + \eta \ln(x/k), \tag{19}$$

where k can vary over households and is used to capture demographic effects.

In conclusion, by using a demand or Engel curve derived from a piglog cost function, we are assured that our macro or market functions have the same desirable properties as the micro functions.