

Appendix A—Equivalence of Counter-Cyclical Payment Rate and Put Option Returns

Let:

ETP = Effective target price

MYAP = Marketing-year average price

NLR = National Loan Rate

Returns to buying a put option with a strike price equal to the effective target price

$$(1a) \quad (ETP - MYAP) \quad \text{or} \quad (1b) \quad 0$$

$$MYAP < ETP \quad \quad \quad MYAP \geq ETP$$

Returns to selling a put option with a strike price equal to the national loan rate

$$(2a) \quad - (NLR - MYAP) \quad \text{or} \quad (2b) \quad 0$$

$$MYAP < NLR \quad \quad \quad MYAP \geq NLR$$

Break (1a) into 2 parts

$$(3a) \quad (ETP - MYAP) \quad (3b) \quad (ETP - MYAP)$$

$$NLR \leq MYAP < ETP \quad \quad \quad MYAP < NLR$$

Break 3b into 2 parts after subtracting and adding NLR

$$(4a) \quad (ETP - NLR) \quad (4b) \quad (NLR - MYAP)$$

$$MYAP < NLR \quad \quad \quad MYAP < NLR$$

Returns to buying and selling the put options $\{(1b) + (3a) + (4a) + (4b)\} + \{(2a) + (2b)\}$

$\{(2a) \text{ and } (4b) \text{ sum to zero}\}$ leaving $\{(1b) + (3a) + (4a)\} + \{(2b)\}$

Returns from (1b), (3a), and (2b) apply when $MYAP \geq NLR$

Returns from (4a) apply when $MYAP \leq NLR$

$$(3a) \quad (ETP - MYAP) \quad \text{or} \quad (1b) \quad 0$$

$$NLR \leq MYAP < ETP \quad \quad \quad MYAP \geq ETP$$

$$(4a) \quad (ETP - NLR) \quad \text{or} \quad (2b) \quad 0$$

$$MYAP \leq NLR \quad \quad \quad MYAP \geq NLR$$

(3a), (4a), and (2b) are combined in one term in (5) and (1b) is separate

Equation (5) below is the same as equation (1) in the text

$$(5) \quad \{ETP - (\text{maximum of } MYAP \text{ and } NLR)\} \quad \text{or} \quad 0$$

$$MYAP < ETP \quad \quad \quad MYAP \geq ETP$$

Appendix B—Option Pricing Procedure Used To Estimate Expected Counter-Cyclical Payment Rates

Equations (1) and (2) are used to estimate expected counter-cyclical payment rates.

$$(1) \quad p_{s,mya} = p_{f,mya} e^{-1/2\sigma_f^2 + \sigma_f z} \quad s = 1, 2, 3, \dots, 10,000$$

$p_{s,mya}$ = a simulated marketing-year average price outcome

$p_{f,mya}$ = a forecasted USDA-WASDE marketing-year average price

σ_f = variability of the natural logarithm of USDA-WASDE forecast outcomes to USDA-WASDE forecasts

z = a random draw from the standard normal probability distribution

(2) Counter-cyclical payment rate =

{(Effective target price) - (larger of: a simulated marketing year average price outcome and national loan rate)} if greater than zero

Otherwise:

$$= 0$$

Equation 1 is used to simulate 10,000 marketing-year average price outcomes for a forecast of the marketing year average price. The outcomes reflect estimated forecast error variability. Equation 2 uses the 10,000 price outcomes to simulate 10,000 counter-cyclical payment rate outcomes at marketing-year end. Equation 2 differs from equation 1 in the text in that it contains a simulated marketing-year average price outcome rather than a USDA, NASS-reported or a WASDE-forecasted marketing-year average price. The 10,000 simulated price outcomes and corresponding payment rate outcomes for a forecasted marketing-year average price represent a sample from all the possible outcomes at marketing-year end as viewed from the date on which the marketing-year average price forecast was made. The average of the payment rate outcomes from the sample estimates the expected counter-cyclical payment rate. The standard deviation of the payment rate outcomes estimates the variability of the expected counter-cyclical payment rate.¹

We estimated the expected frequency (probability) of repaying all of an advance partial payment by counting the number of simulated zero total counter-cyclical payment rates for a forecasted marketing year average price. The expected frequency of repaying all the advance partial payment is the count divided by 10,000.

We also estimated the expected frequency of repaying part or all of the advance partial payment by counting the number of less-than-zero simulated counter-cyclical payment rates. The count divided by 10,000 is the expected frequency of repaying part or all of an advance partial payment.

¹Our procedure for estimating counter-cyclical payment rates differs from the simulation procedure used to solve option pricing models. The simulation procedures used to solve option pricing models simulate entire price paths (for example, all the daily prices) from the current date until option expiration. For example, the Kema and Vorst and the Turnbull and Wakeman simulation procedures for estimating average option prices simulate entire price paths. We did not simulate the entire price path of a time series because there are no reported time series whose average price equals the marketing-year average price.

The average of the 10,000 simulated price outcomes, $p_{s,mya}$, from equation 1 is an unbiased estimate of the forecast, $p_{f,mya}$. We specified equation (1) to produce unbiased estimates of expected marketing-year average prices.²

Simulation procedures similar to the procedure we used are used to solve option models with payments based on an arithmetic average price because, like our model for counter-cyclical payments, they do not have analytical solutions. Analytical approximation methods have been shown to be less accurate for estimating options with payments based on average price than the simulation procedure (James, 2003, pp. 215-216). In addition, the simulation procedure can estimate the variability of the expected counter-cyclical payment rate.

²Adding g to the exponent in equation 3, as follows, $e^{(g-1/2\sigma_f^2)+\sigma_f\epsilon}$ produces biased simulated forecasts when g is not equal to zero. The average forecast error or bias is (e^g-1) times the forecast.

Appendix C—Procedure for Estimating Forecast Error Variability

$$(1) \hat{\mu}_{fe} = (1/25) \sum_{i=1}^{25} \ln(p_{oi} / p_{fi})$$

$$(2) \sigma_{fe} = (1/24) \sqrt{\sum_{i=1}^{25} [\ln(p_{oi} / p_{fi}) - \hat{\mu}_{fe}]^2}$$

where :

i = 1, 2, ..., 25 is for marketing years 1980 through 2004

p_{fi} = marketing - year average price forecast for marketing year i

p_{oi} = marketing - year average price outcome for marketing year i

ln = the natural logarithm

$\hat{\mu}_{fe}$ = average continuous growth rate in the forecast error

σ_{fe} = standard deviation of the continuous growth rate in the forecast error

Appendix D—Determination of Time Value in the Counter-Cyclical Payment Rate

Appendix figure 1 shows the relationship between time value in the counter-cyclical payment rate and the forecasted marketing-year average price.

Time value in the figure was calculated by subtracting the line labeled “USDA method” from the line labeled “Option pricing method (October)” in figure 4. Time values in figures 2, 3, or 5 would work equally well in explaining the relationship between time value and forecasted marketing-year average price.

The time value corresponding to a forecasted marketing-year average price is determined by the potential price moves relative to the forecast. The potential price moves are the potential forecast errors—potential marketing-year average price outcomes minus the forecasted marketing-year average price.

Time value can be estimated by:

1. Averaging the effects of the potential price increases relative to a forecasted marketing-year average price on the counter-cyclical payment rate and subtracting the intrinsic value,¹
2. Averaging the effects of the potential price decreases relative to a forecasted marketing-year average price on the counter-cyclical payment rate and subtracting the intrinsic value, and
3. Averaging time values from steps 1 and 2.²

The following figure and description of estimating the effects of potential price increases and decreases relative to a forecasted price on time value are used to explain the relationship between time value and forecasted price and

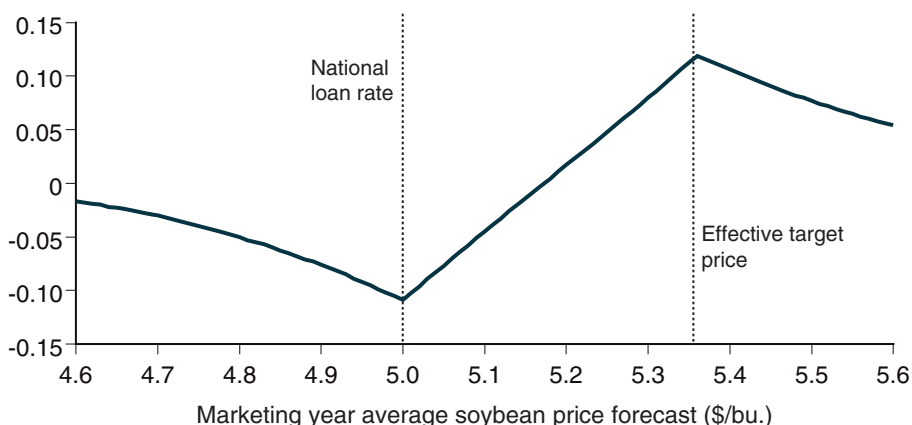
¹Intrinsic value is the counter-cyclical payment rate evaluated at the forecasted marketing-year average price level. The counter-cyclical payment rates for the curves labeled “USDA method” in figures 2 through 5 are intrinsic values.

²The option pricing procedure we used and described in appendix B could be programmed to separate the simulated price movements into positive and negative movements relative to each forecasted price and programmed to use the procedure described in steps 1, 2, and 3 to estimate the counter-cyclical payment rate and time value.

Appendix figure 1

Time value for USDA-WASDE October-soybean marketing year average price forecasts

Time value (\$/bu.)



Source: Compiled by USDA, Economic Research Service using the option pricing model, the USDA method, and the soybean data.

how time value is determined. The explanation uses two levels and three ranges of forecasted marketing-year average price.

1. Forecasted Marketing-Year Average Price equals National Loan Rate (the lower kink in the curve)

Time value is smallest and less than zero. No potential price decreases relative to the national loan rate can increase the counter-cyclical payment rate. All potential price increases relative to the national loan rate decrease the counter-cyclical payment rate.

2. Forecasted Marketing-Year Average Price equals Effective Target Price (the upper kink in the curve)

Time value is largest and greater than zero. All potential price decreases relative to the effective target price increase the counter-cyclical payment rate. No potential price increases decrease the counter-cyclical payment rate.

3. Forecasted Marketing-Year Average Price is greater than zero and less than National Loan Rate (before the lower kink)

As forecasted price increases, the effects of all potential negative price movements relative to forecasted price on the counter-cyclical payment rate remain constant at the maximum counter-cyclical payment rate. As forecasted price increases, the average effect of the potential positive price movements relative to forecasted price decreases the expected counter-cyclical payment rate.

4. Forecasted Marketing-Year Average Price is greater than National Loan Rate and less than Effective Target Price (between the kinks)

As forecasted price increases, the average effect of potential negative price movements on the counter-cyclical payment rate become larger and the average effect of potential positive price movements on the counter-cyclical payment rate become smaller. The net effect is an increase in the counter-cyclical payment rate. The figure shows that as the forecasted marketing-year price increases from the level of the national loan rate, a price level is reached where the average effects of the potential price increases and potential price decreases on the counter-cyclical payment rate balance—resulting in zero time value.

5. Forecasted Marketing-Year Average Price is greater than Effective Target Price (after the upper kink)

As forecasted price increases, the effects of all potential price increases relative to forecasted price on the counter-cyclical payment remain constant at zero while the average effects of the potential price decrease—resulting in smaller expected counter-cyclical payment.

Appendix E—Hedging the Counter-Cyclical Payment Rate With Call Options on Futures Contracts

The following equation shows the returns to hedging counter-cyclical payment rate losses with call options on futures contracts.

(1) hedging returns =

$$\sum_{i=1}^N \{(\text{hedge ratio}_i)(\text{futures call option return}_i)\} - \text{counter-cyclical payment loss}$$

futures call option return_i = return for futures call option contract with expiration (maturity) date i

hedge ratio i = ratio of call option bushels to bushels eligible for counter-cyclical payments or call option i

N = number of futures call option contract expiration dates.

A counter-cyclical payment loss relative to price expectations occurs when the marketing-year average price outcome is larger than the forecasted marketing-year average price and also larger than the national loan rate. The size of a loss equals the smaller of the marketing-year average price outcome and the effective target price minus the larger of the forecasted marketing-year average price and the national loan rate.

Call options on futures contracts provide a payment rate equal to the futures price outcome minus the option strike price at contract maturity if the futures price outcome is larger than the strike price. They can offset counter-cyclical payment rate losses from price expectations to the extent that increases in the marketing-year average price above price expectations are matched by increases in futures price outcomes above price expectations. We assume that a hedge is formed on the day a USDA-WASDE marketing-year average price forecast is made.

Call options on futures contracts in equation 1 have a strike price equal to the current futures price or a strike price equivalent to the national loan rate. A strike price equal to the current futures price is selected if forecasted marketing-year average price is greater than or equal to the national loan rate. A strike price equivalent to the national loan rate is selected when the forecasted marketing-year average price is less than the national loan rate.

Equation 2 shows the procedure for estimating futures prices equivalent (corresponding) to national loan rates.

$$\begin{aligned} (2) \text{fnlr} &= \text{fp} + (\text{nlr} - \text{fmyap})(\Delta \text{fp} / \Delta \text{myap}) \\ &= \text{fp} + (\Delta \text{myap})(\Delta \text{fp} / \Delta \text{myap}) \end{aligned}$$

fnlr = futures price equivalent to national loan rate

fp = current futures price

nlr = national loan rate

fmyap = forecasted marketing-year average price

$\Delta myap = nlr - fmyap$

$\Delta fp / \Delta myap$ = change in futures price corresponding to a one unit change in marketing-year average price (This ratio is estimated by dividing the covariance of marketing-year average price and futures price by the variance of the marketing-year average price, appendix tables E-1, E-3, and E-5.)

We estimated hedging returns using simulated marketing-year average price and futures price outcomes about their price expectations. The price outcomes were simulated using the Cholesky decomposition of the variance-covariance- matrices for the USDA-WASDE marketing-year average price forecast errors and futures price forecast errors in appendix tables E-1, E-3, and E-5. Price expectations include USDA-WASDE marketing average price forecast and corresponding futures prices. We used the simulation procedure to estimate 10,000 sets of price outcomes that matched the correlations, variances and co-variances in appendix tables E-1 through E-6.

Counter-cyclical returns, futures call option returns, and hedging returns were estimated from the simulated prices. About half of the 10,000 counter-cyclical returns are losses from price expectations when forecasted price is above the national loan rate. The fraction decreases as forecasted price decreases below the national loan rate.

Hedging effectiveness was estimated by regressing the counter-cyclical losses on the corresponding futures call option returns. The regression coefficients are optimal hedging ratios—ratio of call option bushels to eligible counter-cyclical bushels that minimize hedging variance in equation 1. Regression R-square is the percent reduction in counter-cyclical payment loss variance.^{1,2}

Hedging effectiveness can also be estimated using the variance of hedging returns and the variance of counter-cyclical losses from equation 1. The optimal hedge ratios are used to calculate the futures call option returns in equation 1. Hedging effectiveness as measured by the percent reduction in counter-cyclical payment loss variance equals:

$(1 - (\text{hedging return variance} / \text{counter-cyclical payment loss variance}))$
times 100.

Hedging effectiveness using this procedure equals the regression R-square discussed previously.

Hedging effectiveness was also measured by the ratio of average call option gain to average counter-cyclical payment rate loss. Call option gain and counter-cyclical payment loss are taken from equation 1.

¹Ederington used regression to estimate hedge ratios that maximize variance reduction from hedging with futures contracts. Stoll and Whaley show how to use regression to estimate hedge ratios that maximize variance reduction for two or more futures contracts in a hedge. The hedge ratios are the regression coefficients.

²Tompkins uses simulation to estimate hedging outcomes for options with payment based on average price outcome. However, Tompkins did not estimate hedge ratios that maximize variance reduction from hedging.

Appendix table E-1

Soybean variance-covariance matrix USDA-WASDE and corresponding futures contract forecast errors, 1977-2003 marketing years

	WASDE	Nov.	Jan.	March	May	July
WASDE	0.38940					
November	0.25686	0.39058				
January	0.35182	0.35813	0.50641			
March	0.53329	0.42797	0.60745	0.97514		
May	0.65858	0.51575	0.73467	1.18115	1.58836	
July	0.69509	0.45071	0.75477	1.12810	1.42190	1.85630

Source: Prepared by USDA, Economic Research Service using WASDE forecast errors and futures price forecast errors.

Appendix table E-2

Soybean correlation matrix for USDA-WASDE and futures contract forecast errors, 1977-2003 marketing years

	WASDE	Nov.	Jan.	March	May	July
WASDE	1.00					
November	0.66	1.00				
January	0.79	0.81	1.00			
March	0.87	0.69	0.86	1.00		
May	0.84	0.65	0.82	0.95	1.00	
July	0.82	0.53	0.78	0.84	0.83	1.00

Source: Prepared by USDA, Economic Research Service using WASDE forecast errors and futures price forecast errors.

Appendix table E-3

Corn variance-covariance matrix for USDA-WASDE and corresponding futures contract forecast errors, 1977-2003 marketing years

	WASDE	Dec.	March	May	July
WASDE	0.05348				
December	0.03327	0.06687			
March	0.05898	0.06322	0.11238		
May	0.08002	0.07291	0.13972	0.20665	
July	0.11416	0.08506	0.15935	0.23792	0.39440

Source: Prepared by USDA, Economic Research Service using WASDE forecast errors and futures price forecast errors.

Appendix table E-4

Corn correlation matrix for USDA-WASDE and futures contract forecast errors, 1977-2003 marketing years

	WASDE	Dec.	March	May	July
WASDE	1.00				
December	0.56	1.00			
March	0.76	0.73	1.00		
May	0.76	0.62	0.92	1.00	
July	0.79	0.52	0.76	0.83	1.00

Source: Prepared by USDA, Economic Research Service using WASDE forecast errors and futures price forecast errors.

Appendix table E-5

Wheat variance-covariance matrix for USDA-WASDE and corresponding futures contract forecast errors, 1977-2003 marketing years

	WASDE	Sept.	Dec.	March	May
WASDE	0.15388				
September	0.10425	0.12850			
December	0.16549	0.12243	0.26359		
March	0.18227	0.11711	0.26915	0.34885	
May	0.20784	0.11686	0.25289	0.33718	0.48623

Source: Prepared by USDA, Economic Research Service using WASDE forecast errors and futures price forecast errors.

Appendix table E-6

Wheat correlation matrix for all USDA-WASDE and futures contract forecast errors, 1977-2003 marketing years

	WASDE	Sept.	Dec.	March	May
WASDE	1.00				
September	0.74	1.00			
December	0.82	0.67	1.00		
March	0.79	0.55	0.89	1.00	
May	0.76	0.47	0.71	0.82	1.00

Source: Prepared by USDA, Economic Research Service using WASDE forecast errors and futures price forecast errors.